Name: ____________________

SM3 13.4 NH: Infinite Geometric Series

Memorize: $S_n = \frac{a_1}{1-r}$

Previously, we discovered the formula for the value of a geometric series, $s = \frac{a_1(1-r^n)}{1-r}$. Today, we're going to sum infinitely many terms which means $n = \infty$. When you add up infinitely many terms, sometimes the sum is divergent (infinitely large). However, other times, the sum is convergent (finite)! When we plug $n = \infty$ into the formula, three different results will occur, depending on the value of r.

If $|r| = 1$, the formula will be $s = \frac{0}{0}$, which is not particularly useful. When $r = 1$, the value of each term remains the same. When you add up infinitely many of the same value, the sum grows each time by the same amount and becomes infinite. Therefore, when $r = 1$, the infinite geometric series diverges.

If $|r| > 1$, the formula will be $s = \frac{a_1(-\infty)}{(1-r)}$, which is equal to ∞. In other words, if you add up infinitely many items that are increasing in value, the sum grows each time by more and more and becomes infinite. Therefore, when $r > 1$, the infinite geometric series diverges.

If $0 < |r| < 1$, the formula will be $s = \frac{a_1}{1-r}$, which is a finite number. Therefore, when $0 < |r| < 1$, the infinite geometric series converges. We'll use this formula to calculate the value of the series.

Example: If the series converges, find the sum of the geometric series. Otherwise, state that it diverges.

$$10 + 20 + 40 + 80 + \cdots$$

Because each term is twice as large as the previous term, $r = 2$. Since $2 > 1$, the series diverges.

$$100 + 50 + 25 + 12.5 + \cdots$$

Because each term is half as large as the previous term, $r = \frac{1}{2}$. Since $\frac{1}{2} < 1$, the series converges.

$$s = \frac{a_1}{1-r}; a_1 = 100, r = \frac{1}{2}$$

$$s = \frac{100}{1-\frac{1}{2}} = \frac{100}{\frac{1}{2}} = 100 \cdot 2 = 200$$

The series converges to 200.

If the series converges, find the sum of the geometric series. Otherwise, state that it diverges.

1) $1 + 2 + 4 + 8 + \cdots$

2) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

3) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$

4) $2 + \frac{4}{5} + \frac{8}{25} + \frac{16}{125} + \cdots$

5) $\sum_{n=1}^{\infty} 3\left(\frac{2}{5}\right)^{n-1}$

6) $\sum_{n=1}^{\infty} 7\left(\frac{4}{3}\right)^{n-1}$

7) $\sum_{n=1}^{\infty} 2\left(-\frac{1}{3}\right)^{n-1}$

8) $\sum_{n=1}^{\infty} \frac{1}{7}\left(\frac{4}{9}\right)^{n-1}$

Write the series using sigma (Σ) notation. Then, if the series converges, find the sum of the geometric series. Otherwise, state that it diverges.

9) $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots$

10) $5 - 25 + 125 - 625 + \cdots$

11) $\frac{25}{64} + \frac{5}{16} + \frac{1}{4} + \frac{1}{5} + \ldots$

12) $\ldots + \frac{1}{25} + \frac{1}{5} + 1 + 5$

Write the repeating decimal as a simplified fraction:

13) $0.\overline{5}$

14) $0.\overline{37}$

15) $0.\overline{111}$

16) $0.\overline{6472}$

17) $1.\overline{58}$

18) $2.\overline{438}$

19) As a brand new driver, Cassie runs into 20 parked cars in her neighborhood at age 16. She only runs into 10 parked cars in her neighborhood while she is 17, perhaps because her skill increases or perhaps because her neighbors are too terrified to leave their cars parked on the street! The pattern continues each year and the number of parked cars she runs into each year is only half of the number of parked cars during the previous year. If she were to live forever, smashing into parked cars until the end of time, with how many parked cars would Cassie have collided?

20) You drop a particularly bouncy ball from a height of 80 feet. The ball is so bouncy, that each time it hits the ground, it returns to a height that is $\frac{3}{4}$ of the most recent height. As the ball is continues bouncing, what is the total distance travelled by the ball approaching? (*hint: the ball moves both up and down, which makes the problem more complicated*)